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INTERNAL SOLITARY WAVES AND SMOOTH BORES WHICH ARE STATIONARY IN A LABORATORY COORDINATE SYSTEM

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Solitary waves and smooth bores are intensively studied in modern hydrodynamics both theoretically and experimentally [1-8]. Laboratory modeling [5-8] has shed definite light on the advantages and drawbacks of the theroetical approaches to the description of these waves. However, some questions remain open. In particular, in the experiments of [5-8] the waves propagated with a slowly varying velocity with respect to an observer, i.e., they were not stationary. There are two possible reasons for this nonstationariness: A stationary wave continued to be formed from the initial disturbance within the experimental apparatus or energy dissipation due to viscosity occurred. It is impossible to determines the nonstationariness in experiments with waves moving with respect to the laboratory coordinate system.

The objective of the present work is to produce and study experimentally solitary waves and smooth bores which are at rest with respect to the laboratory coordinate system. A theoretical analysis, based on [1], of waves on the interface of a shear flow of a two-layer liquid "under a top" showed that stationary waves can exist only if in the initial state both layers move relative to the channel walls.

The experiments were performed on a closed-type laboratory apparatus in whose working channel two layers of immiscible liquids move in the same direction. The apparatus was designed and built on the basis of the theoretically predicted [1] length of the waves under study and the velocities of the layers for which the waves should be at rest. The apparatus (Fig. 1) was 220 cm long and 40 cm high, and its width $B = 15$ cm. It includes a 120 cm long working channel *1* with depth $H = 6$ cm, return channels *2*, deflectors with continuous entrances *3*, a dividing plate *4*, two novel pumps *5* built specially for this apparatus, and a wave generator *6*. The pumps consisted of cylinders rotating around their axes and arranged asymmetrically between the bottom and top of the return channels* [9]. The working liquids were water with density $\rho_1 = 1$ g/cm^3 and kerosene with density $\rho_2 = 0.8$ g/cm^3. The depth h_1 and velocity u_1 of the bottom layer ranged from 1.5 to 2 cm and 20 to 25 cm/sec, respectively, and the depth $h_2 = H - h_1$ and velocity u_2 of the top layer ranged from 4 to 4.5 cm and 15 to 20 cm/sec, respectively. The flow rates Q_i of the liquids in the layers ($i = 1, 2$, where 1 corresponds to the bottom layer and 2 corresponds to the top layer) ranged from 300 to 1200 cm^3/sec. A smooth bore was generated with the help of a barrier, placed at the exit from the working channel. The barrier could be moved vertically into the required position [7].

The waves studied were recorded by making through the liquid still the motion pictures of a matted bright screen located outside the apparatus in order to visualize the interface. The lower liquid was colored with inks. The depths of the layers and the wave amplitudes and profiles were determined from photographs; the measurement error was of the order of 5%. In order to find the velocity of the layers in the working channel, the velocity profiles and flow rates of liquids in the return channels, the velocity profiles and flow rates of liquids in the return channels $q_i = Q_i/B$ were measured as a function of the voltages E_i applied to the pumps by making motion pictures of a slowly sinking solid particle. The experimental data were approximated by power-law functions $q_i = \alpha_i E_i^{n_i}$ (Fig. 2). The curve *1*, describing the flow rate of the bottom layer, was calculated from three calibration the flow rate of the bottom layer, was calculated from three calibration experiments (open circles) with $\alpha_1 = 0.0158$ and $n_1 = 1.6$; the curve *2*, describing the flow rate of the upper layer (kerosene) was calculated from four experiments (black circles) with $\alpha_2 = 0.0221$ and $n_2 = 1.6$. In the main experiments the working voltage E_i was fixed, and the average flow velocities of the layers in the working channel u_1 and u_2 were calculated from the measured depths

*V. I. Bukreev suggested the idea of using pumps with this construction.

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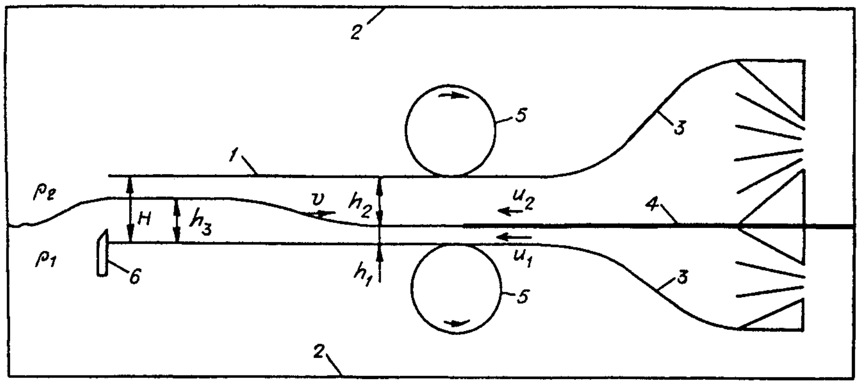

Fig. 1

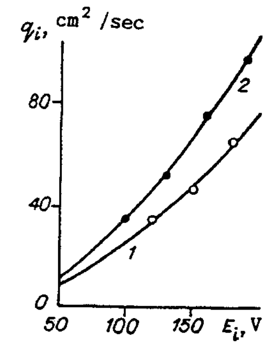

Fig 2

h_1 and h_2 of the undisturbed layers and the flow rates obtained from Fig. 2. The error in measuring the velocity did not exceed 8%.

According to the theroy of [1], for a fixed density ratio $\lambda = \rho_2/\rho_1$ a smooth bore is realized for a definite ratio of the dimensionless velocities of the unperturbed layers $F_i = u_i/\sqrt{gH}$, and the dimensionless depth of the bottom layer behind the wavefront $y_3 = h_3/H$ is determined only by the velocity of the bottom layer:

$$|F_1 - V| + \sqrt{\lambda}\,|F_2 - V| = \sqrt{1 - \lambda}; \tag{1}$$

$$y_3 = (F_1 - V)/\sqrt{1 - \lambda}. \tag{2}$$

Here $V = v/\sqrt{gH}$; v is the velocity of the bore; and, g is the acceleration of gravity. In these experiments $V = 0$, and the simplifie equations (1) and (2) make it possible to find the velocities of the layers and the depth behind the wavefront for a stationary smooth bore. In principle the top layer can also be at rest. But, then if follows fro Eq. (1) with $V = 0$ that $F_1 = \sqrt{1 - \lambda}$, and from Eq. (2) that $V_3 = 1$. For $H = 6$ cm (as in these experiments) $u_1 = 34$ cm/sec, and such a shear flow is unstable with respect to the Kelvin–Helmholtz instability [6, 7]. Here it should be noted that the velocity of the bore does not depend on its amplitude ($y_3 - y_1$) [5], since it does not depend on $y_1 = h_1/H$.

In order to compare to the theory continuous monotonic waves (Fig. 3*a*), which occurred only for layer velocities determined by Eq. (1), were chosen from the series of waves obtained under different flow conditions in the working channel. Bores with undulations or hydraulic jumps (Figs. 3*b* and *c*) were observed when the theoretical and experimental flow parameters were different. The arrows mark the bottom and cover of the working channel. The experimentally recorded waves were compared to theoretical bores according to three indicators: monotonicity of the wave, depth of the bottom layer behind the wavefront, and the profile of the smooth bore. We were able to obtain free stationary smooth bores of a rise in the water level such that the parameters of the bores agreed satisfactorily with the theoretical predictions.

The profile computed according to [1] (solid the *1*) and the experimental profile (open circles) of a smooth bore displayed in Fig. 3*a* with $y_1 = 0.310$, $F_1 = -0.219$, and $F_2 = -0.252$ are compared in Fig. 4*a*. In all experiments $V = 0$ and $\lambda = 0.8$. In Figs. 4 and 6 the dimensionless longitudinal coordinate x/H, measured from the point where $y = (y_1 + y_3)/2$ for the smooth bore, and $y = y_3$ for the solitary wave is plotted along the abscissa and the dimensionless depth $y = h/H$ of the bottom layer, measured from the bottom of the working channel, is plotted along the ordinate. It is evident that the experimental profile is monotonic. The equations (1) and (2) give for the indicated velocities of the layers $V = 0.001$ and $y_3 = 0.494$. The deviation from the experimental velocity is much less than the error with which it is measured. The theoretical depth of the bottom layer behind the wavefront is only 2% less than the experimental value $y_3 = 0.506$. This deviation of y_3 falls within the error of measurement of the depth. The experimental depth of the bottom layer behind the wave ($y_3 = 0.506$) was also used in order to make a detailed comparison of the measured and computed profiles. In this case the computed profile (dashed line *1* and Fig. 4*a*) agrees especially well with the experimentally obtained profile.

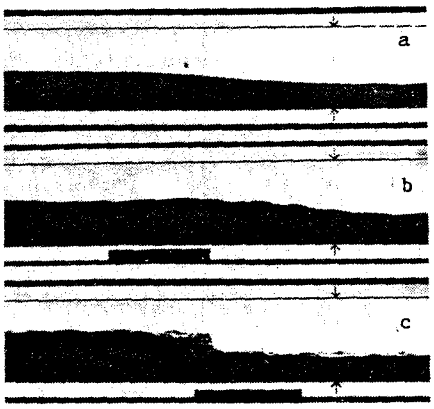

Fig. 3

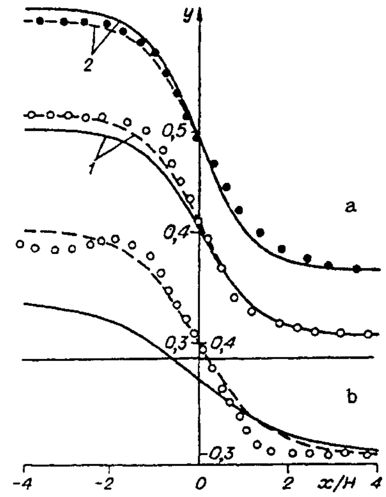

Fig. 4

In the experiments we were able to realize this situation when the velocities of the layers satisfy Eq. (1), and the depth of the bottom liquid the wavefront differs from the value computed from Eq. (2). In this case nonsmooth bores are always observed. Figure 4*b* displays the profile (open dots) of one of the recorded undulational bores with $y_1 = 0.3000$, $F_1 = 0.196$, and $F_2 = -0.281$; according to Eqs. (1) and (2) $V = 0$ and $y_3 = 0.473$. In the experiments the parameter $y_3 = 0.500$ is almost 15% greater than the theroetical value. The computed smooth bore for the indicated parameters is described by the solid line (Fig. 4*b*). The dashed line is the profile computed using the measured depth $y_3 = 0.500$. It is evident that it too differs substantially from the experimental profile.

In order to obtain waves at rest the layers must move with velocities ~20 cm/sec. The surface tension at the water-kerosene prevents the development of the Kelvin–Helmholtz instability up to a difference of 19 cm/sec between the velocities of the layers [6, 7]. Even with a significantly smaller velocity shear, however, the interface is strongly disturbed. For this reason, smooth bores with relatively low amplitude $(h_3 - h_1) \simeq 1.5$ cm and velocity shear $(u_2 - u_1) \simeq 10$ cm/sec were obtained in the experiments. According to Eq. (2), the depth of the bottom layer behind the wavefront is all the larger the higher the velocity F_1 of this layer. As F_1 increases, according to Eq. (1), the velocity F_2 should decrease and the velocity shear $(F_2 - F_2)$ should increase. This intensifies undesirable three-dimensional disturbances of the interface, which provoke the development of a hydraulic jump. Figure 3*c* illustrates such a situation for $y_1 = 0.333$, $F_1 = -0.389$, $F_2 = 0.082$. Here, on the basis of Eqs. (1) and (2) $V = -0.01$ and $y_3 = 0.870$, and in the experiment $y_3 = 0.673$. In the case of a smaller velocity shear smooth bores with short-wavelength (compared to the wavelength of the main wave) disturbances, which can be compared to the theoretical bores, and observed. In Fig. 4a the computed profile (solid line *2*) is compared to the measured profile (dark circles) for $y_1 = 0.366$, $F_1 = -0.269$, $F_2 = -0.199$; Eqs. (1) and (2) give $V = 0$ and $y_3 = 0.601$. The experimentally observed depth $y_3 = 0.590$ differs by only 1.5% from the thoretical depth. The dashed line *2* is the profile computed with the experimental value of the parameter $y_3 = 0.590$.

The amplitude of the moving solitary wave is a free parameter, and according to [1] the wave velocity V and y_3 (the maximum distance from the interface to the bottom of the channel) are related to the relation

$$\frac{(F_1 - V)^2}{\mu y_3} + \frac{(F_2 - V)^2}{\mu (1 - y_3)} = 1, \quad \mu = 1 - \lambda. \tag{3}$$

For the stationary wave $V = 0$. Then the parameter y_3 and together with it the wave amplitude $y_3 - y_1$ also are no longer free. They are determined only by the velocities of the unperturbed layers F_1 and F_2. For this reason, there must be a mechanism that maintains a constant wave amplitude. In the experiments a small obstacle (open squares in Fig. 5) was placed at the bottom of the channel. For certain layer velocities F_1 and F_2 this obstacle generated a solitary wave in the form of a bump and compensated losses to friction owing to the main flow.

In order to compare to the theory continuous, almost symmetric, waves were chosen from the series of waves obtained with different velocities of the layers (Fig. 5*a*). The wave amplitude and profile, which were compared to the experimental amplitude and profile, were calculated from the measured depths and velocities of the unperturbed layers. When the flow conditions

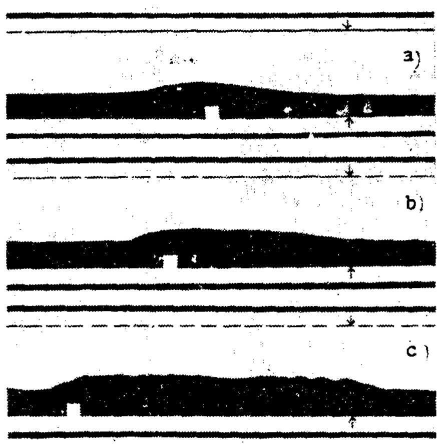

Fig. 5

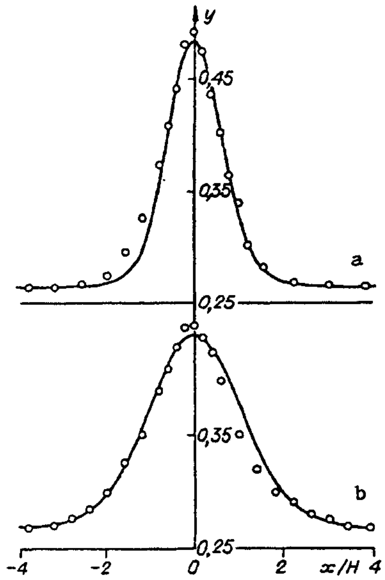

Fig. 6

in the experiment were different from the theoretical flow conditions, significantly asymmetric distrubances or sign-alternating waves are observed in (Figs. 5*b* and *c*).

In Fig. 6*a* the profile of a statinary solitary internal wave (solid line) calculated according to [1] is compared to the experimental profile (dots) with $y_1 = 0.264$, $V = 0$, $F_1 = -0.276$, $F_2 = -0.166$. For such velocities of the layers Eq. (3) gives $y_3 = 0.484$, which is only 1.5% less than the experimentally observed depth $y_3 = 0.491$. It should be noted here that a velocity shear between the layers is not necessary to generate a stationary solitary wave. For this reason, together with wave generation on the shear flow, waves at rest, when the undisturbed layers moved the same velocities, were realized. In Fig. 6*b* one such wave (dots) is compared to calculations (solid line) with $y_1 = 0.268$ and $F_1 = F_2 = -0.233$. For such velocities Eq. (3) gives a depth $y_3 = 0.441$, which is only 2% less than the experimentally observed depth $y_3 = 0.450$. It is evident that the computed profiles agree quite well with the measured profiles.

The wave generation methods employed on these experiments compensated friction losses owing to the main flow, and the smooth bore and solitary wave which were realized had infinite lifetimes (at least, tens of minutes) with constant shape. For this reason, they can be considered to be stationary and it is the viscosity of real liquids that is responsible for the nonstationariness of the waves in the experiments of [5-8].

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